Trigonometric Functions

Exercise 12.8 for Class XI

Question # 1 Show that

(i) $r = 4R \sin\frac{\alpha}{2} \sin\frac{\beta}{2} \sin\frac{\gamma}{2}$ (ii) $s = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

Question # 2 Show that:

$$r = a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\sec\frac{\alpha}{2} = b\sin\frac{\gamma}{2}\sin\frac{\alpha}{2}\sec\frac{\beta}{2} = c\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sec\frac{\gamma}{2}$$

Question # 3 Show that:

(i) $r_1 = 4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ (ii) $r_2 = 4R\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\gamma}{2}$

(iii) $r_3 = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$

Question # 4 Show that:

(i) $r_1 = s\tan\frac{\alpha}{2}$ (ii) $r_2 = s\tan\frac{\beta}{2}$

(iii) $r_3 = s\tan\frac{\gamma}{2}$

Question # 5 Prove that:

(i) $r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2$ (ii) $r r_1 r_2 r_3 = \Delta^2$

(iii) $r_1 + r_2 + r_3 - r = 4R$ (iv) $r_1 r_2 r_3 = r s^2$

Question # 6 Find R, r, r_1, r_2 and r_3, if measures of the sides of triangle ABC are

(i) $a = 13$, $b = 14$, $c = 15$ (ii) $a = 34$, $b = 20$, $c = 42$

Question # 7 Prove that in an equilateral triangle,

(i) $r : R : r_1 = 1 : 2 : 3$ (ii) $r : R : r_1 : r_2 : r_3 = 1 : 2 : 3 : 3 : 3$

Question # 8 Prove that:

(i) $\Delta = r^2 \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$ (ii) $r = s\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(iii) $\Delta = 4Rr\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

Question # 9 Show that

(i) $\frac{1}{2rR} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$ (ii) $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

Question # 10 Prove that:

(i) $r = \dfrac{a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}{\cos\frac{\alpha}{2}}$ (ii) $r = \dfrac{b\sin\frac{\alpha}{2}\sin\frac{\gamma}{2}}{\cos\frac{\beta}{2}}$

Question # 11 Prove that: $abc(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta s$