

Question # 1 $\int \frac{-2x}{\sqrt{4-x^2}} dx$

Solution $\int \frac{-2x}{\sqrt{4-x^2}} dx$

Put $t = 4 - x^2$
 $\Rightarrow dt = -2x dx$

$$= \int \frac{dt}{\sqrt{t}}$$

$$= \int (t)^{-\frac{1}{2}} dt$$

$$= \frac{(t)^{\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{(t)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{t} + c$$

$$= 2\sqrt{4-x^2} + c \quad \underline{Ans.}$$

Question # 2 $\int \frac{dx}{x^2 + 4x + 13}$

Solution

$$\int \frac{dx}{x^2 + 4x + 13}$$

$$= \int \frac{dx}{x^2 + 2(x)(2) + (2)^2 - (2)^2 + 13}$$

$$= \int \frac{dx}{(x+2)^2 - 4 + 13}$$

$$= \int \frac{dx}{(x+2)^2 + 9}$$

$$= \int \frac{dx}{(x+2)^2 + (3)^2}$$

Put $t = x+2$
 $\Rightarrow dt = dx$

$$= \int \frac{dt}{t^2 + 3^2}$$

$$= \frac{1}{3} \operatorname{Tan}^{-1} \frac{t}{3} + c$$

$$= \frac{1}{3} \operatorname{Tan}^{-1} \frac{x+2}{3} + c \quad \underline{Ans.}$$

Question # 3 $\int \frac{x^2}{4+x^2} dx$ **Solution**

$$\begin{aligned}
& \int \frac{x^2}{4+x^2} dx \\
&= \int \left(1 - \frac{4}{4+x^2}\right) dx \\
&= \int dx - 4 \int \frac{dx}{4+x^2} \\
&= x - 4 \int \frac{dx}{(2)^2 + x^2} \\
&= x - 4 \cdot \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + c \\
&= x - 2 \tan^{-1} \left(\frac{x}{2} \right) + c \quad \underline{\text{Ans.}}
\end{aligned}$$

Question # 4 $\int \frac{1}{x \ln x} dx$ **Solution**

$$\begin{aligned}
& \int \frac{1}{x \ln x} dx \\
&= \int \frac{1}{\ln x} \cdot \frac{1}{x} dx \\
&= \int \frac{\frac{1}{x}}{\ln x} dx \quad \text{Put } t = \ln x \\
&= \int \frac{1}{t} dt \quad \Rightarrow \quad dt = \frac{1}{x} dx \\
&= \ln|t| + c \\
&= \ln|\ln x| + c \quad \underline{\text{Ans.}}
\end{aligned}$$

Question # 5 $\int \frac{e^x}{e^x + 3} dx$ **Solution**

$$\begin{aligned}
& \int \frac{e^x}{e^x + 3} dx \quad \text{Put } t = e^x + 3 \\
& \Rightarrow \quad dt = e^x dx \\
&= \int \frac{dt}{t} \\
&= \ln|t| + c \\
&= \ln|e^x + 3| + c \quad \underline{\text{Ans.}}
\end{aligned}$$

Question # 6 $\int \frac{x+b}{(x^2 + 2bx + c)^{\frac{1}{2}}} dx$

Solution

$$\begin{aligned} & \int \frac{x+b}{(x^2 + 2bx + c)^{\frac{1}{2}}} dx \\ & \quad \text{Put } t = x^2 + 2bx + c \\ & \quad \Rightarrow dt = (2x+2b)dx \\ & = \int \frac{\frac{1}{2}dt}{t^{\frac{1}{2}}} \\ & \quad \Rightarrow dt = 2(x+b)dx \\ & = \frac{1}{2} \int t^{-\frac{1}{2}} dt \\ & = \frac{1}{2} \cdot \frac{t^{\frac{1}{2}+1}}{\left(-\frac{1}{2}+1\right)} + c_2 \\ & = \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_2 \\ & = \frac{1}{2} \cdot 2 t^{\frac{1}{2}} + c_2 \\ & = (x^2 + 2bx + c)^{\frac{1}{2}} + c_2 \\ & = \sqrt{x^2 + 2bx + c} + c_2 \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 7 $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$

Solution

$$\begin{aligned} & \int \frac{\sec^2 x}{\sqrt{\tan x}} dx \\ & \quad \text{Put } t = \tan x \\ & \quad \Rightarrow dt = \sec^2 x dx \\ & = \int \frac{dt}{\sqrt{t}} \\ & = \int t^{-\frac{1}{2}} dt \\ & = \frac{t^{\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \\ & = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c \\ & = 2 t^{\frac{1}{2}} + c \\ & = 2(\tan x)^{\frac{1}{2}} + c \end{aligned}$$

$$\begin{aligned} & = 2\sqrt{\tan x} + c \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 8 (a) $\frac{dx}{\sqrt{x^2 - a^2}}$

Solution

$$\frac{dx}{\sqrt{x^2 - a^2}}$$

Put $x = a \sec \theta$
 $\Rightarrow dx = a \sec \theta \tan \theta d\theta$

$$\begin{aligned}
 &= \int \frac{a \sec \theta \tan \theta d\theta}{\sqrt{(a \sec \theta)^2 - a^2}} \\
 &= \int \frac{a \sec \theta \tan \theta d\theta}{\sqrt{a^2 (\sec^2 \theta - 1)}} \\
 &= \int \frac{a \sec \theta \tan \theta d\theta}{\sqrt{a^2 \tan^2 \theta}} \\
 &= \int \frac{a \sec \theta \tan \theta d\theta}{a \tan \theta} \\
 &= \int \sec \theta d\theta \\
 &= \ln |\sec \theta + \tan \theta| + c \\
 &= \ln \left| \sec \theta + \sqrt{\sec^2 \theta - 1} \right| + c \\
 &= \ln \left| \frac{x}{a} + \sqrt{\frac{x^2}{a^2} - 1} \right| + c \\
 &= \ln \left| \frac{x}{a} + \sqrt{\frac{x^2 - a^2}{a^2}} \right| + c \\
 &= \ln \left| \frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right| + c \\
 &= \ln \left| \frac{x + \sqrt{x^2 - a^2}}{a} \right| + c \\
 &= \ln |x + \sqrt{x^2 - a^2}| - \ln a + c \\
 &= \ln |x + \sqrt{x^2 - a^2}| + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

here $c = -\ln a + c$

Question # 8(b) $\int \sqrt{a^2 - x^2} dx$

Solution

$$\begin{aligned}
 & \sqrt{a^2 - x^2} dx \\
 &= \int \sqrt{a^2 - a^2 \sin^2 \theta} \cdot a \cos \theta d\theta && \text{Put } x = a \sin \theta \\
 &= \int \sqrt{a^2 (1 - \sin^2 \theta)} \cdot a \cos \theta d\theta \\
 &= \int \sqrt{a^2 \cos^2 \theta} \cdot a \cos \theta d\theta && \because 1 - \sin^2 \theta = \cos^2 \theta \\
 &= \int a \cos \theta \cdot a \cos \theta d\theta \\
 &= a^2 \int \cos^2 \theta d\theta && \because \cos^2 \theta = \frac{1 + \cos 2\theta}{2} \\
 &= a^2 \int \frac{1 + \cos 2\theta}{2} d\theta \\
 &= \frac{a^2}{2} \int (1 + \cos 2\theta) d\theta \\
 &= \frac{a^2}{2} \left(\theta + \frac{\sin 2\theta}{2} \right) + c \\
 &= \frac{a^2}{2} \left(\theta + \frac{2 \sin \theta \cos \theta}{2} \right) + c \\
 &= \frac{a^2}{2} \left(\theta + \sin \theta \sqrt{1 - \sin^2 \theta} \right) + c && \left| \begin{array}{l} x = a \sin \theta \\ \frac{x}{a} = \sin \theta \\ \sin^{-1} \frac{x}{a} = \theta \end{array} \right. \\
 &= \frac{a^2}{2} \left(\sin^{-1} \frac{x}{a} + \frac{x}{a} \sqrt{1 - \frac{x^2}{a^2}} \right) + c \\
 &= \frac{a^2}{2} \left(\sin^{-1} \frac{x}{a} + \frac{x}{a} \sqrt{\frac{a^2 - x^2}{a^2}} \right) + c \\
 &= \frac{a^2}{2} \left(\sin^{-1} \frac{x}{a} + \frac{x}{a} \frac{\sqrt{a^2 - x^2}}{a} \right) + c \\
 &= \frac{a^2}{2} \left(\sin^{-1} \frac{x}{a} + \frac{x}{a^2} \sqrt{a^2 - x^2} \right) + c \\
 &= \frac{a^2}{2} \sin^{-1} \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

Question # 9 $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$

$$\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$$

$$\begin{aligned}
 &= \int \frac{\sec^2 \theta \, d\theta}{(1+\tan^2 \theta)^{\frac{3}{2}}} && \text{Put } x = \tan \theta \\
 &= \int \frac{\sec^2 \theta \, d\theta}{(\sec^2 \theta)^{\frac{3}{2}}} && \Rightarrow dx = \sec^2 \theta \, d\theta \\
 &= \int \frac{\sec^2 \theta \, d\theta}{\sec^3 \theta} && \because 1 + \tan^2 \theta = \sec^2 \theta \\
 &= \int \frac{d\theta}{\sec \theta} \\
 &= \int \cos \theta \, d\theta \\
 &= \sin \theta + c \\
 &= \frac{\sin \theta}{\cos \theta} \cdot \cos \theta + c \\
 &= \tan \theta \cdot \frac{1}{\sec \theta} + c \\
 &= \tan \theta \cdot \frac{1}{\sqrt{1+\tan^2 \theta}} + c \\
 &= \frac{x}{\sqrt{1+x^2}} + c \quad \underline{\text{Ans.}} && \because x = \tan \theta
 \end{aligned}$$

Question # 10 $\int \frac{1}{(1+x^2) \tan^{-1} x} dx$

Solution

$$\begin{aligned}
 &\int \frac{1}{(1+x^2) \tan^{-1} x} dx \\
 &= \int \frac{1}{\tan^{-1} x} \cdot \frac{1}{(1+x^2)} dx \\
 &= \int \frac{1/(1+x^2)}{\tan^{-1} x} dx
 \end{aligned}$$

$$\begin{aligned}
 &\text{Put } t = \tan^{-1} x \\
 &\Rightarrow dt = \frac{1}{1+x^2} dx
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{1}{t} dt \\
 &= \ln|t| + c \\
 &= \ln|\tan^{-1} x| + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

Question # 11 $\int \sqrt{\frac{1+x}{1-x}} dx$

$$\begin{aligned}
 & \int \sqrt{\frac{1+x}{1-x}} dx \\
 &= \int \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \cdot \cos\theta d\theta \quad \text{Put } x = \sin\theta \\
 &= \int \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta}} \cdot \cos\theta d\theta \\
 &= \int \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} \cdot \cos\theta d\theta \\
 &= \int \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} \cdot \cos\theta d\theta \\
 &= \int \frac{1+\sin\theta}{\cos\theta} \cdot \cos\theta d\theta \\
 &= \int (1+\sin\theta) d\theta \\
 &= \theta - \cos\theta + c \\
 &= \theta - \sqrt{1-\sin^2\theta} + c \\
 &= \sin^{-1}x - \sqrt{1-x^2} + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow dx = \cos\theta d\theta \\
 & \therefore x = \sin\theta \\
 & \therefore \sin^{-1}x = \theta
 \end{aligned}$$

Question # 12 $\int \frac{\sin\theta}{1+\cos^2\theta} d\theta$

Solution

$$\begin{aligned}
 & \int \frac{\sin\theta}{1+\cos^2\theta} d\theta \\
 &= \int \frac{-dt}{1+t^2} = -\int \frac{dt}{1+t^2} \quad \text{Put } t = \cos\theta \\
 &= -\tan^{-1}t + c \\
 &= -\tan^{-1}(\cos\theta) + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

$$\begin{aligned}
 & \Rightarrow dt = -\sin\theta d\theta \\
 & \Rightarrow -dt = \sin\theta d\theta
 \end{aligned}$$

Question # 13 $\int \frac{ax}{\sqrt{a^2-x^4}} dx$

Solution

$$\begin{aligned}
 & \int \frac{ax}{\sqrt{a^2-x^4}} dx \\
 &= a \int \frac{x}{\sqrt{a^2-x^4}} dx
 \end{aligned}$$

$$\text{Put } t = x^2 \text{ then } t^2 = x^4$$

$$\begin{aligned}
 &= a \int \frac{\frac{1}{2}dt}{\sqrt{a^2-t^2}} \quad \Rightarrow \quad dt = 2x dx \\
 &= \frac{a}{2} \int \frac{dt}{\sqrt{a^2-t^2}} \quad \Rightarrow \quad \frac{1}{2}dt = x \cdot dx \\
 &= \frac{a}{2} \sin^{-1}\left(\frac{t}{a}\right) + c \\
 &= \frac{a}{2} \sin^{-1}\left(\frac{x^2}{a}\right) + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

$$\therefore \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a}$$

Question # 14 $\int \frac{dx}{\sqrt{7-6x-x^2}}$

$$\begin{aligned} & \int \frac{dx}{\sqrt{7-6x-x^2}} \\ &= \int \frac{dx}{\sqrt{-\left(x^2+6x-7\right)}} \\ &= \int \frac{dx}{\sqrt{-\left(x^2+2(3)(x)+(3)^2-(3)^2-7\right)}} \\ &= \int \frac{dx}{\sqrt{-\left((x+3)^2-16\right)}} \\ &= \int \frac{dx}{\sqrt{16-(x+3)^2}} \end{aligned}$$

$$\int \frac{dx}{\sqrt{7-6x-x^2}}$$

Put $t = x + 3$
 $\Rightarrow dx = dt$

$$\begin{aligned} &= \int \frac{dt}{\sqrt{16-t^2}} \\ &= \int \frac{dx}{\sqrt{(4)^2-(t)^2}} \\ &= \sin^{-1}\left(\frac{t}{4}\right) + c \\ &= \sin^{-1}\left(\frac{x+3}{4}\right) + c \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 15 $\int \frac{\cos x}{\sin x \cdot \ln \sin x} dx$

Solution

$$\begin{aligned} & \int \frac{\cos x}{\sin x \cdot \ln \sin x} dx \\ &= \int \frac{1}{\ln \sin x} \cdot \frac{\cos x}{\sin x} dx \end{aligned}$$

Put $t = \ln \sin x$
 $\Rightarrow dt = \frac{1}{\sin x} \cdot \cos x dx$

$$\begin{aligned} &= \int \frac{1}{t} dt \\ &= \ln|t| + c \\ &= \ln|\ln \sin x| + c \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 16 $\int \cos x \left(\frac{\ln \sin x}{\sin x} \right) dx$

Solution

$$\begin{aligned} & \int \cos x \left(\frac{\ln \sin x}{\sin x} \right) dx \\ &= \int \ln \sin x \cdot \frac{\cos x}{\sin x} dx \end{aligned}$$

Put $t = \ln \sin x$
 $\Rightarrow dt = \frac{1}{\sin x} \cdot \cos x dx$

$$\begin{aligned} &= \int t \cdot dt \\ &= \frac{t^2}{2} + c \\ &= \frac{(\ln \sin x)^2}{2} + c \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 17 $\int \frac{x dx}{4+2x+x^2}$

Solution

$$\begin{aligned} & \int \frac{x dx}{4+2x+x^2} \\ &= \frac{1}{2} \int \frac{2x dx}{x^2+2x+4} \\ &= \frac{1}{2} \int \frac{(2x+2)-2}{x^2+2x+4} dx \\ &= \frac{1}{2} \int \left(\frac{2x+2}{x^2+2x+4} - \frac{2}{x^2+2x+4} \right) dx \\ &= \frac{1}{2} \int \frac{2x+2}{x^2+2x+4} dx - \frac{1}{2} \int \frac{2}{x^2+2x+4} dx \\ &= \frac{1}{2} \int \frac{2x+2}{x^2+2x+4} dx - \frac{2}{2} \int \frac{dx}{x^2+2x+1+3} \\ &= \frac{1}{2} \int \frac{2x+2}{x^2+2x+4} dx - \int \frac{dx}{(x+1)^2 + (\sqrt{3})^2} \\ &= \frac{1}{2} \ln|x^2+2x+4| - \frac{1}{\sqrt{3}} \operatorname{Tan}^{-1} \frac{x+1}{\sqrt{3}} + c \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 18 $\int \frac{x}{x^4+2x^2+5} dx$

Solution

$$\begin{aligned} & \int \frac{x}{x^4+2x^2+5} dx \quad \text{Put } t = x^2 \quad \text{then } t^2 = x^4 \\ & \qquad \qquad \qquad dt = 2x dx \quad \Rightarrow \quad \frac{1}{2} dt = x dx \\ &= \int \frac{\frac{1}{2} dt}{t^2+2t+5} \\ &= \frac{1}{2} \int \frac{dt}{t^2+2t+1+4} \\ &= \frac{1}{2} \int \frac{dt}{(t+1)^2+(2)^2} \\ &= \frac{1}{2} \cdot \frac{1}{2} \operatorname{Tan}^{-1} \left(\frac{t+1}{2} \right) + c \\ &= \frac{1}{4} \operatorname{Tan}^{-1} \left(\frac{x^2+1}{2} \right) + c \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 19 $\int \left[\cos \left(\sqrt{x} - \frac{x}{2} \right) \right] \times \left(\frac{1}{\sqrt{x}} - 1 \right) dx$

Solution

$$\begin{aligned} & \int \left[\cos \left(\sqrt{x} - \frac{x}{2} \right) \right] \times \left(\frac{1}{\sqrt{x}} - 1 \right) dx \\ & \qquad \qquad \qquad \text{Put } t = \sqrt{x} - \frac{x}{2} \\ & \qquad \qquad \qquad \Rightarrow dt = \left(\frac{1}{2} x^{-\frac{1}{2}} - \frac{1}{2} \right) dx \\ &= \int \cos t \cdot 2 dt \\ &= 2 \int \cos t dt \quad \Rightarrow dt = \frac{1}{2} \left(\frac{1}{\sqrt{x}} - 1 \right) dx \\ &= 2 \sin t + c \quad \Rightarrow 2 dt = \left(\frac{1}{\sqrt{x}} - 1 \right) dx \\ &= 2 \sin \left(\sqrt{x} - \frac{x}{2} \right) + c \quad \underline{\text{Ans.}} \end{aligned}$$

Question # 20 $\int \frac{x+2}{\sqrt{x+3}} dx$

Solution

$$\begin{aligned}
 & \int \frac{x+2}{\sqrt{x+3}} dx \\
 &= \int \frac{x+2+1-1}{\sqrt{x+3}} dx \\
 &= \int \frac{x+3-1}{\sqrt{x+3}} dx \\
 &= \int \left(\frac{x+3}{\sqrt{x+3}} - \frac{1}{\sqrt{x+3}} \right) dx \\
 &= \int \left((x+3)^{1-\frac{1}{2}} - (x+3)^{-\frac{1}{2}} \right) dx \\
 &= \int \left((x+3)^{\frac{1}{2}} - (x+3)^{-\frac{1}{2}} \right) dx \\
 &= \int (x+3)^{\frac{1}{2}} dx - \int (x+3)^{-\frac{1}{2}} dx \\
 &= \int (x+3)^{\frac{1}{2}} \cdot 1 dx - \int (x+3)^{-\frac{1}{2}} \cdot 1 dx \\
 &= \frac{(x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(x+3)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \\
 &= \frac{(x+3)^{\frac{3}{2}}}{3/2} - \frac{(x+3)^{\frac{1}{2}}}{1/2} + c \\
 &= \frac{2(x+3)^{\frac{3}{2}}}{3} - 2\sqrt{x+3} + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

Solution

$$\begin{aligned}
 & \int \frac{x+2}{\sqrt{x+3}} dx \\
 & \qquad \text{Put } t = x+3 \text{ then } x = t-3 \\
 & \qquad \Rightarrow dt = dx \\
 &= \int \frac{t-3+2}{\sqrt{t}} dt \\
 &= \int \frac{t-1}{(t)^{\frac{1}{2}}} dt \\
 &= \int \left(\frac{t}{(t)^{\frac{1}{2}}} - \frac{1}{(t)^{\frac{1}{2}}} \right) dt \\
 &= \int \left((t)^{\frac{1}{2}} - (t)^{-\frac{1}{2}} \right) dt \\
 &= \frac{(t)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(t)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \\
 &= \frac{(t)^{\frac{3}{2}}}{3/2} - \frac{(t)^{\frac{1}{2}}}{1/2} + c \\
 &= \frac{2(x+3)^{\frac{3}{2}}}{3} - 2(x+3)^{\frac{1}{2}} + c \\
 &= \frac{2(x+3)^{\frac{3}{2}}}{3} - 2\sqrt{x+3} + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

Question # 21 $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$

Solution

$$\begin{aligned}
 & \int \frac{\sqrt{2}}{\sin x + \cos x} dx \\
 &= \int \frac{1}{\frac{1}{\sqrt{2}}(\sin x + \cos x)} dx \\
 &= \int \frac{1}{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x} dx \\
 &\quad \because \frac{1}{\sqrt{2}} = \sin \frac{\pi}{4} = \cos \frac{\pi}{4} \\
 &= \int \frac{1}{\sin \frac{\pi}{4} \cdot \sin x + \cos \frac{\pi}{4} \cdot \cos x} dx \\
 &= \int \frac{1}{\cos\left(x - \frac{\pi}{4}\right)} dx \\
 &= \int \sec\left(x - \frac{\pi}{4}\right) dx \\
 &= \ln \left| \sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right) \right| + c \quad \underline{\text{Ans.}}
 \end{aligned}$$

Question # 22 $\int \frac{dx}{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x}$

Solution

$$\begin{aligned}
 & \int \frac{dx}{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x} \\
 &\quad \because \cos \frac{\pi}{3} = \frac{1}{2} \quad \& \quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \\
 &= \int \frac{dx}{\frac{\cos \frac{\pi}{3}}{\sin \frac{\pi}{3}} \sin x + \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} \cos x} \\
 &= \int \frac{dx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} \\
 &= \int \cosec\left(x + \frac{\pi}{3}\right) dx \\
 &= \ln \left| \cosec\left(x + \frac{\pi}{3}\right) - \cot\left(x + \frac{\pi}{3}\right) \right| + c
 \end{aligned}$$