Question # 1 (i) $\int\left(3x^2-2x+1\right)dx$

Solution

$$\int\left(3x^2-2x+1\right)dx$$
$$=3\int x^2dx-2\int xdx+\int dx$$
$$=3\cdot\frac{x^{2+1}}{2+1}-2\cdot\frac{x^{1+1}}{1+1}+x+c$$
$$=3\cdot\frac{x^3}{3}-2\cdot\frac{x^2}{2}+x+c$$
$$=x^3-x^2+x+c \qquad \underline{Ans}.$$

Question # 1 (ii) $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

Solution

$$\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$$
$$=\int\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx$$
$$=\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$$
$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$
$$=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c$$
$$=\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+c \quad \underline{Ans}.$$

Question # 1 (iii) $\int x\left(\sqrt{x}+1\right)dx$

Solution

$$\int x\left(\sqrt{x}+1\right)dx$$
$$=\int x\left(x^{\frac{1}{2}}+1\right)dx$$
$$=\int\left(x^{\frac{3}{2}}+x\right)dx$$
$$=\int x^{\frac{3}{2}}dx+\int x\,dx$$
$$=\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+\frac{x^{1+1}}{1+1}+c$$
$$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+\frac{x^2}{2}+c$$
$$=\frac{2}{5}x^{\frac{5}{2}}+\frac{1}{2}x^2+c \quad \underline{Ans}.$$

Question # 1 (iv) $\int (2x+3)^{\frac{1}{2}}\,dx$

Solution

$$\int (2x+3)^{\frac{1}{2}}\,dx$$

$$=\frac{(2x+3)^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)\cdot 2}+c$$

$$=\frac{(2x+3)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)\cdot 2}+c$$

$$=\frac{1}{3}(2x+3)^{\frac{3}{2}}+c \quad \underline{Ans}.$$

Question # 1 (v) $\int \left(\sqrt{x}+1\right)^2 dx$

Solution

$$\int \left(\sqrt{x}+1\right)^2 dx$$

$$=\int \left((\sqrt{x})^2+2\sqrt{x}+1\right)dx$$

$$=\int \left(x+2(x)^{\frac{1}{2}}+1\right)dx$$

$$=\int x\,dx+2\int (x)^{\frac{1}{2}}\,dx+\int dx$$

$$=\frac{x^{1+1}}{1+1}+2\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+x+c$$

$$=\frac{x^2}{2}+2\frac{x^{\frac{3}{2}}}{3/2}+x+c$$

$$=\frac{x^2}{2}+\frac{4x^{\frac{3}{2}}}{3}+x+c \quad \underline{Ans}.$$

Question # 1 (vi) $\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

Solution

$$\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$=\int \left(\left(\sqrt{x}\right)^2+\frac{1}{\left(\sqrt{x}\right)^2}-2\sqrt{x}\cdot\frac{1}{\sqrt{x}}\right)dx$$

$$=\int \left(x+\frac{1}{x}-2\right)dx$$

$$=\int x\,dx+\int \frac{1}{x}\,dx-2\int dx$$

$$=\frac{x^2}{2}+\ln|x|-2x+c \quad \underline{Ans}.$$

Question # 1(vii) $\int \frac{3x+2}{\sqrt{x}}dx$

Solution

$$\int \frac{3x+2}{\sqrt{x}}dx$$

$$=\int \frac{3x+2}{x^{1/2}}dx$$

$$=\int \frac{3x}{x^{1/2}}+\frac{2}{x^{1/2}}dx$$

$$=\int \left(3x^{1/2}+2x^{-1/2}\right)dx$$

$$=3\int x^{1/2}dx+2\int x^{-1/2}dx$$

$$=3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+2\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$

$$=3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+c$$

$$=3\cdot\frac{2}{3}\cdot x^{\frac{3}{2}}+2\cdot\frac{1}{2}\cdot x^{\frac{1}{2}}+c$$

$$=2x^{\frac{3}{2}}+x^{\frac{1}{2}}+c \qquad \underline{Ans}.$$

Question # 1(viii) $\int \frac{\sqrt{y}(y+1)}{y}dy$

Solution

$$\int \frac{\sqrt{y}(y+1)}{y}dy$$

$$=\int \frac{(y+1)}{\sqrt{y}}dy$$

$$=\int \left(\frac{y}{\sqrt{y}}+\frac{1}{\sqrt{y}}\right)dy$$

$$=\int \left(y^{\frac{1}{2}}+y^{-\frac{1}{2}}\right)dy$$

$$=\int y^{\frac{1}{2}}dy+\int y^{-\frac{1}{2}}dy$$

$$=\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}+\frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$

$$=\frac{y^{\frac{3}{2}}}{\frac{3}{2}}+\frac{y^{\frac{1}{2}}}{\frac{1}{2}}+c$$

$$=\frac{2}{3}y^{\frac{3}{2}}+2y^{\frac{1}{2}}+c \qquad \underline{Ans}.$$

Question # 1(ix) $\int \frac{\left(\sqrt{\theta}-1\right)^2}{\sqrt{\theta}}\, d\theta$

Solution

$$\int \frac{\left(\sqrt{\theta}-1\right)^2}{\sqrt{\theta}}\, d\theta$$

$$= \int \frac{\theta - 2\sqrt{\theta}+1}{\sqrt{\theta}}\, d\theta$$

$$= \int \left(\frac{\theta}{\sqrt{\theta}} - \frac{2\sqrt{\theta}}{\sqrt{\theta}} + \frac{1}{\sqrt{\theta}} \right) d\theta$$

$$= \int \left(\theta^{\frac{1}{2}} - 2 + \theta^{-\frac{1}{2}} \right) d\theta$$

$$= \frac{\theta^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 2\theta + \frac{\theta^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{\theta^{\frac{3}{2}}}{3/2} - 2\theta + \frac{\theta^{\frac{1}{2}}}{1/2} + c$$

$$= \frac{2}{3}\theta^{\frac{3}{2}} - 2\theta + 2\theta^{\frac{1}{2}} + c \quad \underline{Ans}.$$

Question # 1 (x) $\int \frac{\left(1-\sqrt{x}\right)^2}{\sqrt{x}}\, dx$

Solution

$$\int \frac{\left(1-\sqrt{x}\right)^2}{\sqrt{x}}\, dx$$

$$= \int \frac{1 - 2\sqrt{x}+x}{\sqrt{x}}\, dx$$

$$= \int \left(\frac{1}{\sqrt{x}} - \frac{2\sqrt{x}}{\sqrt{x}} + \frac{x}{\sqrt{x}} \right) dx$$

$$= \int \left(x^{-\frac{1}{2}} - 2 + x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 2x + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= \frac{x^{\frac{1}{2}}}{1/2} - 2x + \frac{x^{\frac{3}{2}}}{3/2} + c$$

$$= 2x^{\frac{1}{2}} - 2x + \frac{2}{3}x^{\frac{3}{2}} + c \quad \underline{Ans}.$$

Question # 1 (xi) $\int \frac{e^{2x}+e^x}{e^x}\, dx$

Solution

$$\int \frac{e^{2x}+e^x}{e^x}\, dx$$

$$= \int \left(\frac{e^{2x}}{e^x} + \frac{e^x}{e^x} \right) dx$$

$$= \int \left(e^x + 1 \right) dx$$

$$= \int e^x dx + \int dx$$

$$= e^x + x + c \quad \underline{Ans}.$$

Question # 2 (i) $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$

Solution

$$\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$$

$$=\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}\cdot\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$=\int \frac{\sqrt{x+a}-\sqrt{x+b}}{x+a-x-b}\,dx$$

$$=\int \frac{(x+a)^{\frac{1}{2}}-(x+b)^{\frac{1}{2}}}{a-b}\,dx$$

$$=\frac{1}{a-b}\left[\int (x+a)^{\frac{1}{2}}\,dx-\int (x+b)^{\frac{1}{2}}dx\right]$$

$$=\frac{1}{a-b}\left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{(x+b)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]+c$$

$$=\frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]+c$$

$$=\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}}-(x+b)^{\frac{3}{2}}\right]+c \quad \underline{Ans}.$$

Question # 2 (ii) $\int \frac{1-x^2}{1+x^2}\,dx$

Solution

$$\int \frac{1-x^2}{1+x^2}\,dx$$

$$=\int\left(-1+\frac{2}{1+x^2}\right)dx$$

$$=-\int dx+2\int \frac{1}{1+x^2}\,dx$$

$$=-x+2Tan^{-1}x+c \quad \underline{Ans}.$$

$$\begin{array}{r|l} & -1 \\ \hline 1+x^2 & 1-x^2 \\ & -1-x^2 \\ & \overset{+}{}\ \overset{+}{} \\ \hline & 2 \end{array}$$

Question # 2 (iii) $\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}$

Solution

$$\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}$$

$$=\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}\cdot\frac{\sqrt{x+a}-\sqrt{x}}{\sqrt{x+a}-\sqrt{x}}$$

$$=\int \frac{\sqrt{x+a}-\sqrt{x}}{x+a-x}\,dx$$

$$=\int \frac{(x+a)^{\frac{1}{2}}-(x)^{\frac{1}{2}}}{a}\,dx$$

$$=\frac{1}{a}\left[\int (x+a)^{\frac{1}{2}}\,dx-\int (x)^{\frac{1}{2}}dx\right]$$

$$=\frac{1}{a}\left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{(x)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]+c$$

$$=\frac{1}{a}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x)^{\frac{3}{2}}}{\frac{3}{2}}\right]+c$$

$$=\frac{2}{3a}\left[(x+a)^{\frac{3}{2}}-x^{\frac{3}{2}}\right]+c \quad \underline{Ans}.$$

Question # 2 (iv) $\int (a-2x)^{\frac{3}{2}}\,dx$

Solution

$$\int (a-2x)^{\frac{3}{2}}\,dx$$

$$=-\frac{1}{2}\int (a-2x)^{\frac{3}{2}}\,(-2)\,dx$$

$$=-\frac{1}{2}\cdot\frac{(a-2x)^{\frac{3}{2}+1}}{\left(\frac{3}{2}+1\right)}+c$$

$$=\frac{(a-2x)^{\frac{5}{2}}}{\left(\frac{5}{2}\right)\cdot(-2)}+c$$

$$=-\frac{(a-2x)^{\frac{5}{2}}}{5}+c \qquad \underline{Ans}.$$

Question # 2 (v) $\int \frac{\left(1+e^{x}\right)^{3}}{e^{x}}\,dx$

Solution

$$\int \frac{\left(1+e^{x}\right)^{3}}{e^{x}}\,dx$$

$$=\int \frac{\left(1+3e^{x}+3e^{2x}+e^{3x}\right)}{e^{x}}\,dx$$

$$=\int \left(\frac{1}{e^{x}}+\frac{3e^{x}}{e^{x}}+\frac{3e^{2x}}{e^{x}}+\frac{e^{3x}}{e^{x}}\right)dx$$

$$=\int \left(e^{-x}+3+3e^{x}+e^{2x}\right)dx$$

$$=\frac{e^{-x}}{-1}+3x+3e^{x}+\frac{e^{2x}}{2}+c$$

$$=-e^{-x}+3x+3e^{x}+\frac{e^{2x}}{2}+c \qquad \underline{Ans}.$$

Question # 2 (vi) $\int \sin(a+b)x\,dx$

Solution

$$\int \sin(a+b)x\,dx$$

$$=\frac{1}{a+b}\int \sin(a+b)x\cdot(a+b)\,dx$$

$$=\frac{1}{a+b}\left(-\cos(a+b)x\right)+c$$

$$=-\frac{\cos(a+b)x}{a+b}+c \qquad \underline{Ans}.$$

Question # 2 (vii) $\int \sqrt{1-\cos 2x}\,dx$

Solution

$$\int \sqrt{1-\cos 2x}\,dx$$

$$=\int \sqrt{2\sin^{2} x}\,dx \qquad \because\ \sin^{2} x=\frac{1-\cos 2x}{2}$$

$$=\sqrt{2}\int \sin x\,dx$$

$$=\sqrt{2}\left(-\cos x\right)+c$$

$$=-\sqrt{2}\cos x+c \qquad \underline{Ans}.$$

Question # 2 (viii) $\int \ln x \times \frac{1}{x}\, dx$

Solution

$$\int \ln x \times \frac{1}{x}\, dx$$

Put $t = \ln x$

$\Rightarrow dt = \frac{1}{x} dx$

$$= \int t \quad dt$$

$$= \frac{t^2}{2} + c$$

$$= \frac{(\ln x)^2}{2} + c \qquad \underline{Ans}.$$

Question # 2 (ix) $\int \sin^2 x\, dx$

Solution

$$\int \sin^2 x\, dx$$

$$= \int \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$= \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right) dx$$

$$= \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x\, dx$$

$$= \frac{1}{2} x - \frac{1}{2} \frac{\sin 2x}{2} + c$$

$$= \frac{1}{2} x - \frac{1}{4} \sin 2x + c \qquad \underline{Ans}.$$

Question # 2 (x) $\int \frac{1}{1 + \cos x} dx$

Solution

$$\int \frac{1}{1 + \cos x} dx$$

$$= \int \frac{1}{2 \cos^2 \frac{x}{2}} dx \qquad \because \cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$

$$= \frac{1}{2} \int \sec^2 \frac{x}{2} dx$$

$$= \frac{1}{2} \frac{\tan \frac{x}{2}}{1/2} + c$$

$$= \tan \frac{x}{2} + c \qquad \underline{Ans}.$$

Solution

$$\int \frac{1}{1 + \cos x} dx$$

$$= \int \frac{1}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x} dx$$

$$= \int \frac{1 - \cos x}{1 - \cos^2 x} dx$$

$$= \int \frac{1 - \cos x}{\sin^2 x} dx$$

$$= \int \left(\frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x} \right) dx$$

$$= \int \left(\operatorname{cosec}^2 x - \frac{\cos x}{\sin x \cdot \sin x} \right) dx$$

$$= \int \operatorname{cosec}^2 x dx - \int \operatorname{cosec} x \cot x\, dx$$

$$= -\cot x - (-\operatorname{cosec} x) + c$$

$$= \operatorname{cosec} x - \cot x + c \qquad \underline{Ans}.$$

Question # 2(xi) $\int \frac{ax+b}{ax^2+2bx+c}dx$

Solution

$$\int \frac{ax+b}{ax^2+2bx+c}dx$$

Put $f(x)=ax^2+2bx+c$

$\Rightarrow f'(x)=2ax+2b$

$\Rightarrow f'(x)=2(ax+b)$

$\Rightarrow \frac{1}{2}f'(x)=ax+b$

$$=\int \frac{\frac{1}{2}f'(x)}{f(x)}dx$$

$$=\frac{1}{2}\int \frac{f'(x)}{f(x)}dx \quad =\frac{1}{2}\ln|f(x)|+c_1$$

$$=\frac{1}{2}\ln\left|ax^2+2bx+c\right|+c_1$$

Question # 2 (xii) $\int \cos 3x \sin 2x\, dx$

Solution

$$\int \cos 3x \sin 2x\, dx$$

$$=\frac{1}{2}\int 2\cos 3x \sin 2x\, dx$$

$$=\frac{1}{2}\int [\sin(3x+2x)-\sin(3x-2x)]\, dx$$

$$=\frac{1}{2}\int [\sin 5x-\sin x]\, dx$$

$$=\frac{1}{2}\left[-\frac{\cos 5x}{5}-(-\cos x)\right]+c$$

$$=-\frac{1}{2}\left[\frac{\cos 5x}{5}-\cos x\right]+c \qquad \underline{Ans}.$$

Question # 2 (xiii) $\int \frac{\cos 2x-1}{1+\cos 2x}\, dx$

Solution

$$\int \frac{\cos 2x-1}{1+\cos 2x}\, dx$$

$$=-\int \frac{1-\cos 2x}{1+\cos 2x}\, dx \qquad \because \sin^2 x=\frac{1-\cos 2x}{2}$$

$$=-\int \frac{2\sin^2 x}{2\cos^2 x}\, dx \qquad \cos^2 x=\frac{1+\cos 2x}{2}$$

$$=-\int \tan^2 x\, dx$$

$$=-\int (\sec^2 x-1)\, dx$$

$$=-\int \sec^2 x\, dx+\int dx$$

$$=-\tan x+x+c \qquad \underline{Ans}.$$

Question # 2 (xiv) $\int \tan^2 x\, dx$

Solution

$$\int \tan^2 x\, dx$$

$$=\int (\sec^2 x-1)\, dx$$

$$=\int \sec^2 x\, dx-\int dx$$

$$=\tan x-x+c \qquad \underline{Ans}.$$
