Find δx and dy in the following cases

Question # 1(i) $y = x^2 - 1$

Solution

$$y = x^2 - 1 \;\ldots\ldots\; (i)$$

$$x = 3 \;\; \& \;\; \delta x = 3.02 - 3 = 0.02$$

$$y + \delta y = (x + \delta x)^2 - 1$$

$$\Rightarrow \;\; \delta y = (x + \delta x)^2 - 1 - x^2 + 1$$

$$= (x + \delta x)^2 - x^2$$

Put $x = 3$ & $\delta x = 0.02$

$$\delta y = (3 + 0.02)^2 - (3)^2 \;\Rightarrow\; \boxed{\delta y = 0.1204}$$

Taking differential of (i)

$$dy = d\left(x^2 - 1\right)$$

$$\Rightarrow \;\; dy = 2x\, dx$$

Put $x = 3$ & $dx = \delta x = 0.02$

$$dy = 2(3)(0.02) \quad \Rightarrow \quad \boxed{dy = 0.12}$$

Question # 1(ii) $y = x^2 + 2x$

Solution

Do yourself as above.

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Question # 1(iii) $y=\sqrt{x}$

Solution

$$y=\sqrt{x} \;=\; x^{\frac{1}{2}} \;\ldots\ldots.\,(i)$$

$$x=4 \quad \& \quad \delta x=4.41-4=0.41$$

$$y+\delta y=(x+\delta x)^{\frac{1}{2}}$$

$$\Rightarrow \quad \delta y=(x+\delta x)^{\frac{1}{2}}-x^{\frac{1}{2}}$$

Put $x=4$ & $\delta x=0.41$

$$\delta y=(4+0.41)^{\frac{1}{2}}-(4)^{\frac{1}{2}} \;=2.1-2 \quad \Rightarrow \quad \boxed{\delta y=0.1}$$

Taking differential of (i)

$$dy=\frac{d}{dx}\left(x^{\frac{1}{2}}\right)dx$$

$$=\frac{1}{2}x^{-\frac{1}{2}}\,dx \;=\; \frac{1}{2x^{\frac{1}{2}}}\,dx$$

Put $x=4$ & $dx=\delta x=0.41$

$$dy \;=\; \frac{1}{2(4)^{\frac{1}{2}}}(0.41) \;=\; \frac{0.41}{4} \quad \Rightarrow \quad \boxed{dy=0.1025}$$

Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations.

Question # 2(i) $xy+x \;=\; 4$

Solution

$$xy+x \;=\; 4$$

Taking differential on both sides

$$d(xy)+dx \;=\; d(4)$$

$$\Rightarrow \quad xdy+ydx+dx \;=\; 0$$

$$\Rightarrow \quad xdy+(y+1)dx \;=\; 0$$

$$\Rightarrow \quad xdy \;=\; -(y+1)dx$$

$$\Rightarrow \quad \frac{dy}{dx} \;=\; -\frac{y+1}{x} \quad \& \quad \frac{dx}{dy}=-\frac{x}{y+1}$$

Question # 2(ii) $x^2+2y^2 = 16$

Do yourself as above

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Question # 2(iii) $x^4 + y^2 = xy^2$

Solution

$$x^4 + y^2 = xy^2$$

Taking differential

$$d\left(x^4\right) + d\left(y^2\right) = d\left(xy^2\right)$$

$$\Rightarrow 4x^3dx + 2ydy = x \cdot 2ydy + y^2dx$$

$$\Rightarrow 2ydy - 2xydy = y^2dx - 4x^3dx$$

$$\Rightarrow 2y(1-x)dy = \left(y^2 - 4x^3\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - 4x^3}{2y(1-x)} \quad \& \quad \frac{dx}{dy} = \frac{2y(1-x)}{y^2 - 4x^3}$$

Question # 2(iv) $xy - \ln x = c$

Solution

$$xy - \ln x = c$$

Taking differential

$$d(xy) - d(\ln x) = d(c)$$

$$\Rightarrow xdy + ydx - \frac{1}{x}dx = 0$$

$$\Rightarrow xdy = \frac{1}{x}dx - ydx$$

$$= \left(\frac{1}{x} - y\right)dx$$

$$\Rightarrow xdy = \left(\frac{1-xy}{x}\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-xy}{x^2} \quad \& \quad \frac{dx}{dy} = \frac{x^2}{1-xy}$$

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Use differentials to approximate the values of

Question # 3(i) $\sqrt[4]{17}$

Solution

Let $y = f(x) = \sqrt[4]{x}$

where $x = 16$ and $\delta x = dx = 1$

Taking differential of above

$$dy = d\left(\sqrt[4]{x}\right)$$

$$= d(x)^{\frac{1}{4}}$$

$$= \frac{1}{4}x^{\frac{1}{4}-1}dx = \frac{1}{4}x^{-\frac{3}{4}}dx = \frac{1}{4x^{\frac{3}{4}}}dx$$

Put $x = 16$ and $dx = 1$

$$dy = \frac{1}{4(16)^{\frac{3}{4}}}(1)$$

$$= \frac{1}{4\left(2^4\right)^{\frac{3}{4}}} = \frac{1}{4(8)} = 0.03125$$

Now $f\left(x + dx\right) \approx y + dy$

$$= f(x) + dy \quad \because \ y = f(x)$$

$$\Rightarrow \sqrt[4]{16+1} \approx \sqrt[4]{16} + 0.03125$$

$$\Rightarrow \sqrt[4]{17} \approx \left(2^4\right)^{\frac{1}{4}} + 0.03125 = 2 + 0.03125 = 2.03125$$

Question # 3(ii) $(8.02)^{\frac{1}{3}}$

Solution

Let $y = f(x) = (x)^{\frac{1}{3}}$

Where $x = 8$ & $\delta x = dx = 0.2$

Taking differential of above

$$dy = d(x)^{\frac{1}{3}}$$

$$= \frac{1}{3}(x)^{-\frac{2}{3}}dx = \frac{1}{3x^{\frac{2}{3}}}dx$$

Put $x = 8$ and $dx = 0.2$

$$dy = \frac{1}{3(8)^{\frac{2}{3}}}(0.2) = \frac{1}{3\left(2^3\right)^{\frac{2}{3}}}(0.2) = \frac{1}{3(4)}(0.2) = 0.01667$$

Now $f\left(x + \delta x\right) \approx y + dy$

$$= f(x) + dy \quad \because \ y = f(x)$$

$$\Rightarrow (8+0.2)^{\frac{1}{3}} = (8)^{\frac{1}{3}} + 0.01667$$

$$\Rightarrow (8.02)^{3} = 2 + 0.01667$$

$$= 2.01667$$

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Question # 3(iii) $31^{\frac{1}{5}}$

Let $y = f(x) = x^{\frac{1}{5}}$

Where $x = 32$ & $\delta x = dx = -1$

Try yourself as above.

Question # 3(iv) $\cos 29^\circ$

Solution

Let $y = f(x) = \cos x$

Where $x = 30^\circ$ & $\delta x = -1^\circ = -\frac{\pi}{180}$ rad $= -0.01745$ rad

Now $dy = d(\cos x)$

$= -\sin x \, dx$

Put $x = 30^\circ$ and $dx = \delta x = -0.01745$

$dy = -\sin 30^\circ(-0.01745) = -(0.5)(-0.01745) = 0.008725$

Now $f(x+\delta x) \approx y + dy$

$= f(x) + dy$

$\Rightarrow \cos(30-1) = \cos 30^\circ + 0.008725$

$\Rightarrow \cos 29^\circ = 0.866 + 0.008725$

$= 0.8747$

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Question # 3(v) $\sin 61^\circ$

Solution

Let $y = f(x) = \sin x$

Where $x = 60^\circ$ & $\delta x = 1^\circ = \dfrac{\pi}{180}\text{rad} = 0.01745\text{ rad}$

Now $dy = d(\sin x) = \cos x\, dx$

Put $x = 60^\circ$ and $dx = \delta x = 0.01745$

$$dy = \cos 60^\circ (0.01745) = (0.5)(0.01745) = 0.008725$$

Now $f(x+\delta x) \approx y + dy$

$$= f(x) + dy$$

$$\Rightarrow \sin(60+1) = \sin 60^\circ + 0.008725$$

$$\Rightarrow \sin 61^\circ = 0.866 + 0.008725 = 0.8747$$

Question # 4 Find the approximate increase in the volume of a cube if the length of its each edge changes from 5 to 5.02.

Solution

Let x be the length of side of cube where

$x = 5$ & $\delta x = 5.02 - 5 = 0.02$

Assume V denotes the volume of the cube.

Then $V = x \cdot x \cdot x = x^3$

Taking differential

$$dV = 3x^2 dx$$

Put $x = 5$ & $dx = \delta x = 0.02$

$$dV = 3(5)^2(0.02) = 1.5$$

Hence increase in volume is 1.5 cubic unit.

Question # 5 Find the approximate increase in the area of circular disc if its diameter is increased from 44cm to 44.4cm.

Solution

Let x denotes diameter of a disc

Where $x = 44\text{ cm}$ & $\delta x = 44.4 - 44 = 0.4$

Then radius $= \dfrac{x}{2}$

Let A denotes the area of the disc

Then $A = \pi(\text{radius})^2$

$$= \pi\left(\frac{x}{2}\right)^2 = \frac{\pi}{4}x^2$$

Taking differential

$$dA = d\left(\frac{\pi}{4}x^2\right) = \frac{\pi}{4}\cdot 2x \cdot dx = \frac{\pi}{2}x\, dx$$

Put $x = 44$ and $dx = \delta x = 0.4$

$$dA = \frac{\pi}{2}(44)(0.4) = (3.14)(22)(0.4) = 27.65$$

Hence change in area is 27.65 cm^2

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