Question # 1 Solve the following triangle ABC in which:

$b=95, \qquad c=34, \qquad \alpha=52^\circ.$

Solution

$b=95, \qquad c=34, \qquad \alpha=52^\circ.$

By law of cosine

$$a^2=b^2+c^2-2bc\cos\alpha$$
$$=(95)^2+(34)^2-2(95)(34)\cos 52^\circ$$
$$=9025+1156-6460(0.6157)$$
$$=6203.578$$
$$\Rightarrow \quad a=\sqrt{6203.578}$$
$$\boxed{a=78.76}$$

$$a^2=b^2+c^2-2bc\cos\alpha$$
$$b^2=c^2+a^2-2ca\cos\beta$$
$$c^2=a^2+b^2-2ab\cos\gamma$$

Again by law of cosine

$$b^2=c^2+a^2-2ca\cos\beta$$
$$\Rightarrow \quad \cos\beta=\frac{c^2+a^2-b^2}{2ca}$$
$$=\frac{(34)^2+(78.76)^2-(95)^2}{2(34)(78.76)}$$
$$=\frac{1156+6203.138-9025}{5355.68}$$
$$=-\frac{1665.862}{5355.68}$$
$$=-0.311$$
$$\Rightarrow \quad \beta=\cos^{-1}(-0.311)$$
$$\boxed{\beta=108^\circ 7' 20''}$$

Since in any triangle

$$\alpha+\beta+\gamma=180$$

$$\alpha+\beta+\gamma=180$$
$$\Rightarrow \quad \gamma=180-\alpha-\beta$$
$$=180^\circ-52^\circ-108^\circ 7' 20''$$
$$\boxed{\gamma=19^\circ 52' 40''}$$

Question # 2 Solve the following triangle ABC in which:

$$b = 12.5 \quad , \quad c = 23 \quad , \quad \alpha = 38^\circ 20'$$

Solution

$$b = 12.5 \quad , \quad c = 23 \quad , \quad \alpha = 38^\circ 20'$$

By law of cosine

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$= (12.5)^2 + (23)^2 - 2(12.5)(23)\cos 38^\circ 20'$$
$$= 156.25 + 529 - 575(0.7844)$$
$$= 234.21$$
$$\Rightarrow \quad a = \sqrt{234.21}$$
$$\boxed{a = 15.304}$$

Again by law of cosine

$$\cos\beta = \frac{c^2 + a^2 - b^2}{2ca}$$
$$b^2 = c^2 + a^2 - 2ca\cos\beta$$
$$= \frac{(23)^2 + (15.304)^2 - (12.5)^2}{2(23)(15.304)}$$
$$= \frac{529 + 234.21 - 156.25}{703.984}$$
$$= \frac{606.96}{703.984}$$
$$= 0.8622$$
$$\beta = \cos^{-1}(0.8622)$$
$$\Rightarrow \quad \boxed{\beta = 30^\circ 26'}$$

Since in any triangle

$$\alpha + \beta + \gamma = 180$$
$$\Rightarrow \quad \gamma = 180 - \alpha - \beta$$
$$= 180^\circ - 38^\circ 20' - 30^\circ 26'$$
$$\boxed{\gamma = 111^\circ 14'}$$

Question # 3 Solve the following triangle ABC in which:

$$a=\sqrt{3}-1=0.732 \quad , \quad b=\sqrt{3}+1=2.732 \quad , \quad \gamma=60^\circ$$

Solution

By law of cosine

$$\begin{aligned} c^2 &= a^2+b^2-ab\cos\gamma \\ &= (0.732)^2+(2.732)^2-2(0.732)(2.732)\cos 60^\circ \\ &= 0.5358+7.4638-1.9998 \\ &= 5.9998 \end{aligned}$$

$$\Rightarrow \quad c=\sqrt{5.9998}$$

$$\Rightarrow \quad \boxed{c=2.449}$$

Again by law of cosines

$$\begin{aligned} \cos\alpha &= \frac{b^2+c^2-a^2}{2bc} \\ &= \frac{(2.732)^2+(2.449)^2-(0.732)}{2(2.732)(2.449)} \\ &= \frac{7.4638+5.9976-0.5358}{13.3813} \\ &= \frac{12.9256}{13.3813} \end{aligned}$$

$$\cos\alpha = 0.9659$$

$$\Rightarrow \quad \alpha=\cos^{-1}(0.9659)$$

$$\Rightarrow \quad \boxed{\alpha=15^\circ}$$

Since in any triangle

$$\alpha+\beta+\gamma=180$$

$$\begin{aligned} \Rightarrow \quad \beta &= 180-\alpha-\gamma \\ &= 180-15-60 \end{aligned}$$

$$\Rightarrow \quad \boxed{\beta=105^\circ}$$

Question # 4 Solve the following triangle ABC in which:

$a = 3 \quad , \quad b = 6 \quad , \quad \gamma = 36^\circ 20'$

Solution

as above

$\downarrow$

Question # 5 Solve the following triangle ABC in which:

$a = 7 \quad , \quad b = 3 \quad , \quad \gamma = 38^\circ 13'$

Solution

Do yourself as above

$\downarrow$

Question # 6 Solve the following triangle, using first law of tangent and then law of sines:

$$a = 36.21 \quad , \quad b = 42.09 \quad , \quad \gamma = 44^\circ 29'$$

Solution

Since $\alpha + \beta + \gamma = 180$

$$\Rightarrow \alpha + \beta = 180 - \gamma$$
$$= 180 - 44^\circ 29'$$
$$\Rightarrow \alpha + \beta = 135^\circ 31' \quad \ldots\ldots\ldots\ldots \text{(i)}$$

By law of tangent

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$$

$$\frac{36.21-42.09}{36.21+42.09} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{135^\circ 31'}{2}\right)}$$

$$\Rightarrow \quad \frac{-5.88}{78.3} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(67^\circ 45'\right)} \quad \Rightarrow -0.0751 = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{2.4443}$$

$$\Rightarrow \quad \tan\left(\frac{\alpha-\beta}{2}\right) = -0.0751(2.4443) = -0.1836$$

$$\Rightarrow \quad \frac{\alpha-\beta}{2} = \tan^{-1}(-0.1836)$$

$$\Rightarrow \quad \frac{\alpha-\beta}{2} = -10^\circ 24' \quad \Rightarrow \alpha - \beta = -20^\circ 48' \quad \ldots\ldots\ldots\ldots \text{(ii)}$$

Adding (i) & (ii)

$$\begin{aligned} \alpha + \beta &= 135^\circ 31' \\ \alpha - \beta &= -20^\circ 48' \\ \hline 2\alpha &= 114^\circ 43' \end{aligned} \quad \Rightarrow \boxed{\alpha = 57^\circ 22'}$$

Putting value of α in eq. (i)

$$57^\circ 22' + \beta = 135^\circ 22'$$
$$\Rightarrow \beta = 135^\circ 22' - 57^\circ 22' \quad \Rightarrow \boxed{\beta = 78^\circ 9'}$$

Now by law of sine

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \quad \Rightarrow \frac{c}{\sin 44^\circ 29'} = \frac{36.21}{\sin 57^\circ 22'}$$

$$\Rightarrow c = \frac{36.21}{\sin 57^\circ 22'} \cdot \sin 44^\circ 29'$$
$$= \frac{36.21}{0.8421} \cdot 0.7007 \quad \Rightarrow \boxed{c = 30.13}$$

Question # 7, 8 & 9 Solve the following triangle, using first law of tangent and then law of sines:

(7) $a = 93,\quad c = 101,\quad \alpha = 80^\circ$ (8) $b = 14.8,\ c = 16.1,\quad \alpha = 42^\circ 45'$

(9) $a = 319,\ b = 168,\quad \alpha = 110^\circ 22'$

Solution

$\downarrow$ *as above*

Question # 10 Solve the following triangle, using first law of tangent and then law of sines:

$b=61 \quad , \quad c=32 \quad , \quad \alpha=59^\circ 30'$

Solution

$b=61 \quad , \quad c=32 \quad , \quad \alpha=59^\circ 30'$

Since $\alpha+\beta+\gamma=180$

$\Rightarrow \beta+\gamma=180-\alpha$

$=180-59^\circ 30'$

$\Rightarrow \beta+\gamma=120^\circ 30'$ ..... (i)

By law of tangent

$$\frac{b-c}{b+c}=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)} \Rightarrow \frac{61-32}{61+32}=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{120^\circ 30'}{2}\right)}$$

$$\Rightarrow \frac{29}{93}=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(60^\circ 15'\right)} \Rightarrow 0.3118=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{1.7496}$$

$$\Rightarrow \tan\left(\frac{\beta-\gamma}{2}\right)=0.3118(1.7496)$$

$$=0.5455$$

$$\Rightarrow \frac{\beta-\gamma}{2}=\tan^{-1}(0.5455)$$

$$\Rightarrow \frac{\beta-\gamma}{2}=28^\circ 37' \qquad \Rightarrow \beta-\gamma=57^\circ 14' \text{ ..... (ii)}$$

Adding (i) & (ii)

$$\begin{aligned} \beta+\gamma &= 120^\circ 30' \\ \beta-\gamma &= 57^\circ 14' \\ \hline 2\beta &= 177^\circ 44' \end{aligned} \qquad \Rightarrow \boxed{\beta=88^\circ 52'}$$

Putting value of α in eq. (i)

$88^\circ 52'+\gamma=120^\circ 30'$

$\Rightarrow \gamma=120^\circ 30'-88^\circ 52' \qquad \Rightarrow \boxed{\gamma=31^\circ 38'}$

Now by law of sine

$$\frac{c}{\sin\gamma}=\frac{a}{\sin\alpha} \qquad \Rightarrow \frac{32}{\sin 31^\circ 38'}=\frac{a}{\sin 59^\circ 30'}$$

$$\Rightarrow c=\frac{32}{\sin 31^\circ 38'}\cdot\sin 59^\circ 30'$$

$$=\frac{32}{0.5244}\cdot 0.8616 \qquad \Rightarrow \boxed{c=52.57}$$

Question # 11 Measure of two sides of the triangle are in the ratio $3:2$ and they include an angle of measure 57° . Find the remaining two angles.

Solution

Let two sides of the triangle are a and b

i.e. $a:b=3:2$ and $\gamma=57^\circ$

$$\frac{a}{b}=\frac{3}{2} \Rightarrow a=\frac{3}{2}b$$

Since $\alpha+\beta+\gamma=180$

$$\alpha+\beta=180-\gamma$$

$$=180-57 \quad \Rightarrow \alpha+\beta=123^\circ \text{ ..... (i)}$$

By law of tangent

$$\frac{a-b}{a+b}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)}$$

$$\Rightarrow \frac{\frac{3}{2}b-b}{\frac{3}{2}b+b}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{123^\circ}{2}\right)}$$

$$\Rightarrow \frac{\frac{1}{2}b}{\frac{5}{2}b}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(61^\circ 30'\right)}$$

$$\Rightarrow \frac{1}{5}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{1.8418}$$

$$\Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right)=\frac{1}{5}(1.8418)=0.3684$$

$$\Rightarrow \frac{\alpha-\beta}{2}=\tan^{-1}(0.3684)=20^\circ 13' \Rightarrow \alpha-\beta=40^\circ 26' \text{ ..... (ii)}$$

Adding (i) & (ii)

$$\begin{aligned} \alpha+\beta &= 123^\circ \\ \alpha-\beta &= 40^\circ 27' \\ \hline 2\alpha &= 163^\circ 27' \end{aligned} \quad \Rightarrow \boxed{\alpha=81^\circ 44'}$$

Putting value of α in eq. (i)

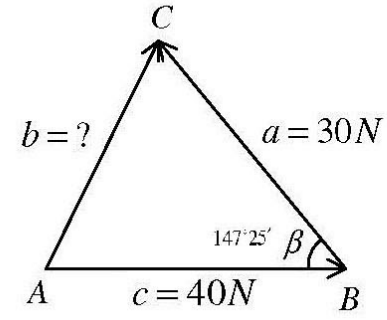
$$81^\circ 44'+\beta=123^\circ$$

$$\Rightarrow \beta=123^\circ-81^\circ 44' \quad \Rightarrow \boxed{\beta=41^\circ 16'}$$

remaining two angles are $\alpha=81^\circ 44'$ & $\beta=41^\circ 16'$

Question # 12 Two forces of 40N and 30N are represented by $\overrightarrow{AB}$ and $\overrightarrow{BC}$ which are inclined at an angle of $147^\circ 25'$. Find $\overrightarrow{AC}$, the resultant of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

Solution



Since $\overrightarrow{AB} = c = 40N$

$\overrightarrow{BC} = a = 30N$

$m\angle B = \beta = 147^\circ 25'$

$\overrightarrow{AC} = b = ?$

By law of cosine

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$
$$= (40)^2 + (30)^2 - 2(40)(30)\cos 147^\circ 25'$$
$$= 1600 + 900 - 2400(-0.8426)$$
$$= 4522.26$$
$$= \sqrt{4522.26}$$

$\Rightarrow \quad b = 67.248$

$\overrightarrow{AC} = 67.248N$