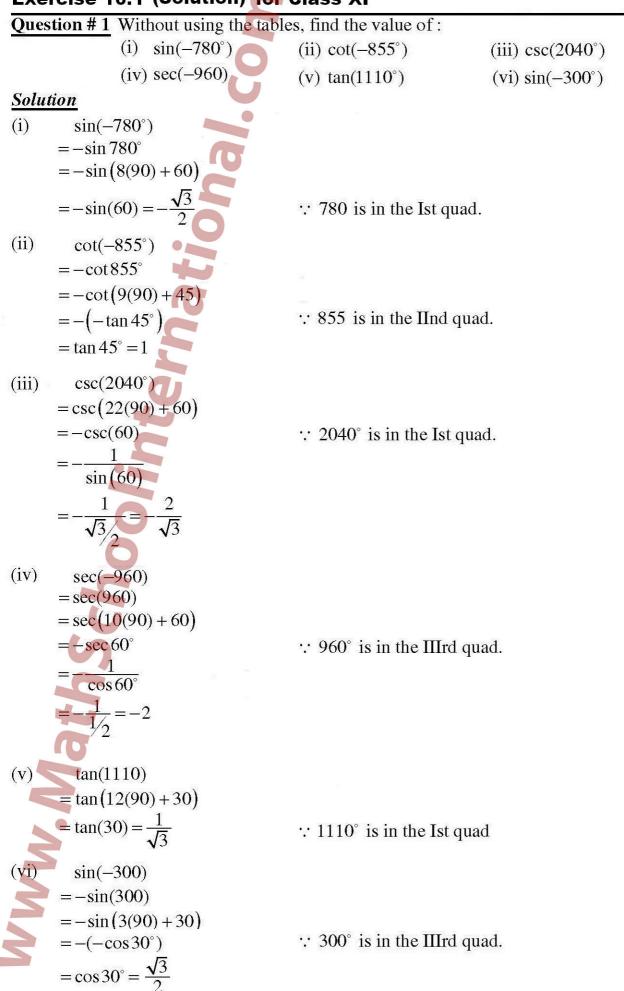
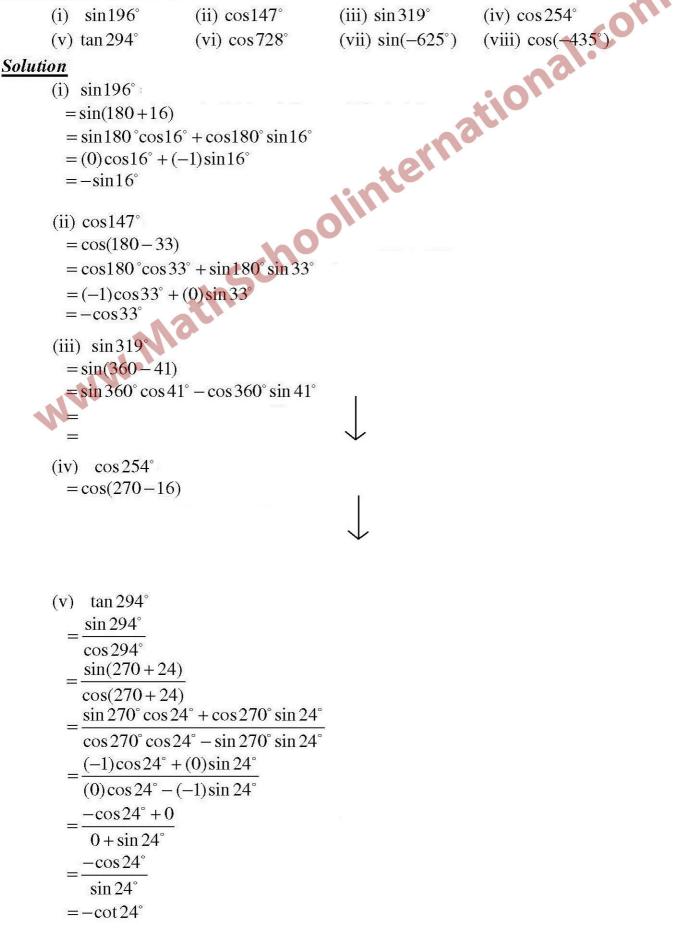
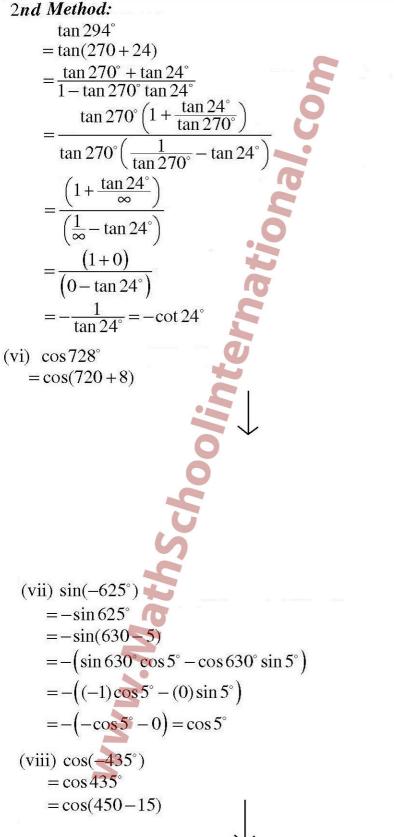
## **Trigonometric Identities** Exercise 10.1 (Solution) for Class XI



## Question # 2

Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45°.





**Question #3** Prove the following:  
(i) 
$$\sin(180 + \alpha)\sin(90 - \alpha) = -\sin\alpha\cos\alpha$$
 (iii)  $\sin 306^{\circ} + \cos 234^{\circ} + \cos 162^{\circ} + \cos 18^{\circ} = 0$   
(ii)  $\sin 780^{\circ} \sin 480^{\circ} + \cos 120^{\circ} \sin 30^{\circ} = \frac{1}{2}$  (iv)  $\cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} = -1$   
**Solution**  
(i) L.I.I.S  
 $= \sin(180 + \alpha)\sin(90 - \alpha)$   
 $= (\sin 180^{\circ} \cos\alpha + \cos 180^{\circ} \sin\alpha))(\sin 90^{\circ} \cos\alpha - \cos 90^{\circ} \sin\alpha)$   
 $= ((0)\cos\alpha + (-1)\sin\alpha))((1)\cos\alpha - (0)\sin\alpha)$   
 $= (0 - \sin\alpha))(\cos\alpha - 0)$   
 $= -\sin\alpha\cos\alpha = R.H.S$   
(ii) L.H.S  
 $= \sin 780^{\circ} \sin 480^{\circ} + \cos 120^{\circ} \sin 30^{\circ}$   
 $= \sin(2\times 360 + 60)\sin(5\times 90 + 30) + \cos(90 + 30)\sin 30^{\circ}$   
 $= \sin 60^{\circ} \cos 30 + (-\sin 30)\sin 30^{\circ}$   
 $= (\sqrt{3})(\sqrt{3}) + (-\frac{1}{2})(\frac{1}{2})$   
 $= \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = R.H.S$   
(iii) L.H.S  
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$   
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$   
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$   
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$   
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$   
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(270 - 36) + \sin 270\cos 36 + \sin 270\cos 36 + \cos 180^{\circ} \cos 18 + \sin 180^{\circ} \sin 36^{\circ} + \cos 28^{\circ} + \cos 28^{\circ} + \cos 28^{\circ} + \cos 18^{\circ}$   
 $= \cos(270 + 36) + \cos(270 - 36) + (-\sin 38) + \cos 18^{\circ}$   
 $= \cos(330^{\circ} \sin 56) - \cos(18^{\circ} + \cos 18^{\circ} + \cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 30^{\circ} + \sin 30^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 330^{\circ} \sin 160^{\circ} + (-\sin 30)^{\circ} (\cos 60^{\circ})$   
 $= (\cos 30^{\circ})(-\sin 60) + (-\sin 30)^{\circ} (\cos 60^{\circ})$   
 $= (\cos 30^{\circ})(-\sin 60) + (-\sin 30)^{\circ} (\cos 60^{\circ})$   
 $= (\cos 30^{\circ})(-\sin 60) + (-\sin 30)^{\circ} (\cos 60^{\circ})$   
 $= (\frac{\sqrt{3}}{2}) \left[-\frac{\sqrt{3}}{2}\right] + (\frac{1}{2}) \left(\frac{1}{2}\right)$  ( $\frac{\sqrt{3}}{2}$ ,  $\frac{\sqrt{3}}{2}\right] + (\frac{\sqrt{3}}{2}\right] + (\frac{\sqrt{3}}{2}\right]$   
 $= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{$ 

Question # 4 Prove that  

$$\frac{\sin^2(\pi + \theta) \tan\left(\frac{3\pi}{2} + \theta\right)}{\sin^2\left(\frac{3\pi}{2} - \theta\right)\cos^2(\pi - \theta)\csc(2\pi - \theta)} = \cos\theta$$
(i) L.H.S

$$=\frac{\sin^2(\pi+\theta) \tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\csc(2\pi-\theta)}$$

First we calculate

$$\sin(\pi+\theta) = \sin\pi\cos\theta + \cos\pi\sin\theta$$
$$= (0)\cos\theta + (-1)\sin\theta$$
$$= 0 - \sin\theta = -\sin\theta$$
$$\Rightarrow \sin^{2}(\pi+\theta) = (-\sin\theta)^{2}$$
$$\tan\left(\frac{3\pi}{2}+\theta\right) = \tan\left(3\cdot\frac{\pi}{2}+\theta\right) = \cot\theta \quad \because \quad \frac{3\pi}{2}+\theta \text{ is in the IVth quad}$$
$$\cot\left(\frac{3\pi}{2}-\theta\right) = \cot\left(3\cdot\frac{\pi}{2}-\theta\right) = \tan\theta \quad \because \quad \frac{3\pi}{2}-\theta \text{ is in the IIIrd quad}$$
$$\Rightarrow \cot^{2}\left(\frac{3\pi}{2}-\theta\right) = (\tan\theta)$$
$$\csc(2\pi-\theta) = \csc(-\theta) = -\csc\theta$$
$$\cos(\pi-\theta) = \cos\pi\cos\theta + \sin\pi\sin\theta$$
$$= (-1)\cos\theta + (0)\sin\theta$$
$$= -\cos\theta + 0 = -\cos\theta$$
$$\Rightarrow \cos^{2}(\pi-\theta) = (-\cos\theta)^{2}$$
$$= \frac{(-\sin\theta)^{2}(-\cot\theta)}{(\tan\theta)^{2}(-\cos\theta)^{2}(-\csc\theta)}$$
$$= \frac{\sin^{2}\theta}{(\cos^{2}\theta)}\frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^{2}\theta}{\cos^{2}\theta}\frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta\cos\theta}{\sin\theta}$$
$$= \cos\theta = \text{R.H.S}$$

$$\frac{\text{Question # 4}}{\text{(ii)}} \frac{\text{Prove that}}{\sec(90^\circ + \theta) \sec(-\theta) \tan(180^\circ - \theta)}$$

$$\frac{\cos(90^\circ + \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)}{\sec(360^\circ - \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)} = -1$$

$$\frac{\text{Solution}}{(\text{ii) L.H.S}}$$

$$= \frac{\cos(90^\circ + \theta) \sec(-\theta) \tan(180^\circ - \theta)}{\sec(360^\circ - \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)}$$

First we calculate

 $\cos(90 + \theta) = -\sin\theta$   $\because 90 + \theta \text{ is in the IInd quad.}$  $\sec(-\theta) = \sec\theta$ 

 $\tan(180 - \theta) = \tan(2(90) - \theta) = -\tan\theta$  $\therefore 180 - \theta$  is in the IInd quad.

 $\sec(360 - \theta) = \sec(-\theta) = \sec\theta$ 

 $\sin(180 + \theta) = \sin(2(90) + \theta) = -\sin\theta$  $\therefore 180 + \theta$  is in the IIIrd quad.

 $\cot(90 - \theta) = \tan \theta$ ::  $90 - \theta$  is in the 1st quad.

 $=\frac{(-\sin\theta)\sec\theta(-\tan\theta)}{\sec\theta(-\sin\theta)(-\tan\theta)}$ 

=1 = R.H.S

= 0 = R.H.S

**Question #5** If  $\alpha, \beta, \gamma$  are the angles of a triangle ABC, then prove that  $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$  $\sin(\alpha + \beta) = \sin \gamma$ (ii) (i)  $\tan(\alpha + \beta) + \tan \gamma = 0$ (iii)  $\cos(\alpha + \beta) = \cos \gamma$ (iv) Since  $\alpha$ ,  $\beta$  and  $\gamma$  are angels of triangle therefore Solution  $\alpha + \beta + \gamma = 180$ (i)L.H.S =  $sin(\alpha + \beta)$  $\alpha + \beta = 180 - \gamma$  $=\sin(180-\gamma)$  $= \sin 180 \cos \gamma - \cos 180 \sin \gamma$  $=(0)\cos\gamma-(-1)\sin\gamma$  $=0+\sin\gamma$  $=\sin \gamma = R.H.S$ L.H.S =  $\cos\left(\frac{\alpha+\beta}{2}\right)$ (ii)Since  $\alpha$ ,  $\beta$  and  $\gamma$  are angels of triangle therefore  $\Rightarrow \frac{\alpha + \beta + \gamma = 180}{\alpha + \beta = 180 - \gamma} \Rightarrow \frac{\alpha + \beta}{2} = \frac{180 - \gamma}{2}$  $=\cos\left(\frac{180-\gamma}{2}\right)$  $=\cos\left(\frac{180}{2}-\frac{\gamma}{2}\right)$  $=\cos\left(90-\frac{\gamma}{2}\right)$  $=\cos 90\cos \frac{\gamma}{2} + \sin 90\sin \frac{\gamma}{2}$  $= (0)\cos\frac{\gamma}{2} + (1)\sin\frac{\gamma}{2}$  $=0+\sin\frac{\gamma}{2}$ Since  $\alpha$ ,  $\beta$  and  $\gamma$  are angels of triangle therefore  $=\sin\frac{\gamma}{2}$  = R.H.S  $\alpha + \beta + \gamma = 180 \implies \alpha + \beta = 180 - \gamma$ L.H.S =  $\cos(\alpha + \beta)$ (iii)  $=\cos(180-\gamma)$  $=\cos 180\cos \gamma + \sin 180\sin \gamma$  $=(-1)\cos\gamma+(0)\sin\gamma$  $= -\cos \gamma + 0$  $=-\cos\gamma = R.H.S$ L.H.S =  $tan(\alpha + \beta) + tan \gamma$ (iv) $= \tan(180 - \gamma) + \tan \gamma$  $=\frac{\tan 180 - \tan \gamma}{1 + \tan 180 \tan \gamma} + \tan \gamma$  $=\frac{(0)-\tan\gamma}{1+(0)\tan\gamma}+\tan\gamma$ Since  $\alpha$ ,  $\beta$  and  $\gamma$  are angels of triangle therefore  $\alpha + \beta + \gamma = 180 \implies \alpha + \beta = 180 - \gamma$  $=\frac{-\tan\gamma}{1+0}+\tan\gamma$  $= -\tan \gamma + \tan \gamma$