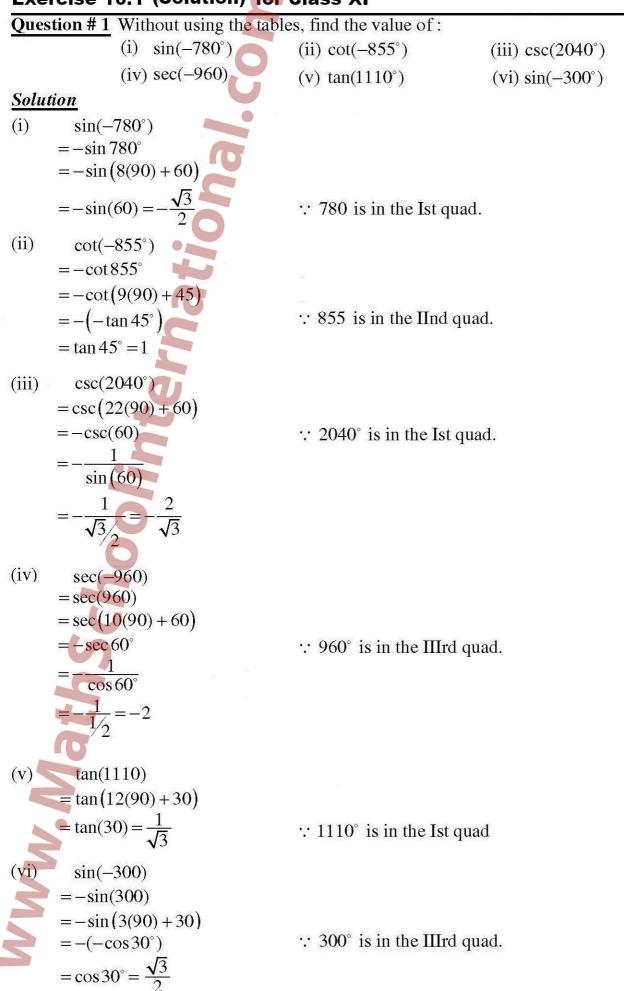
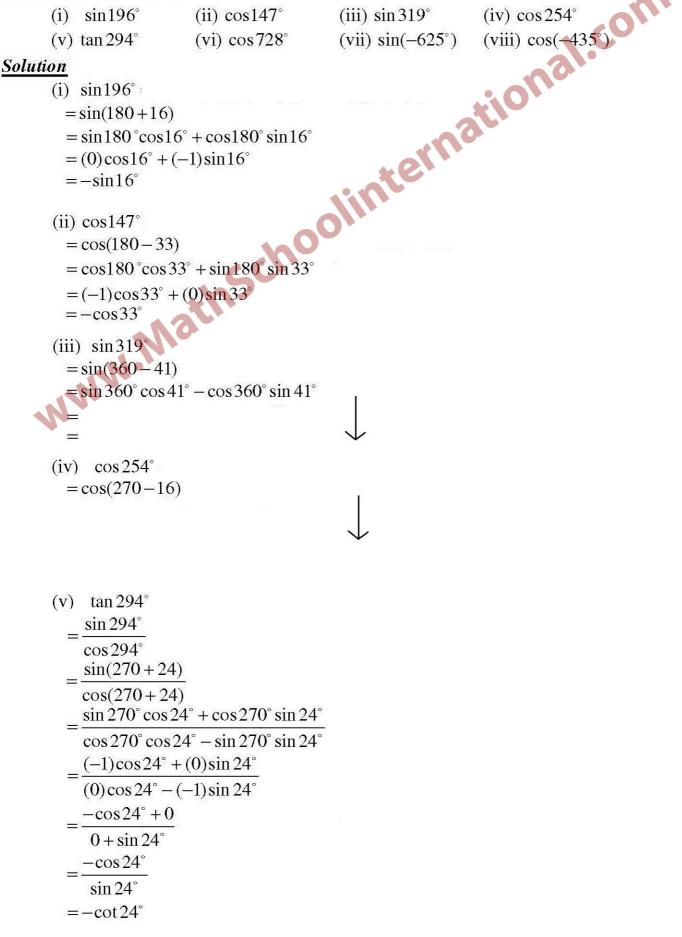
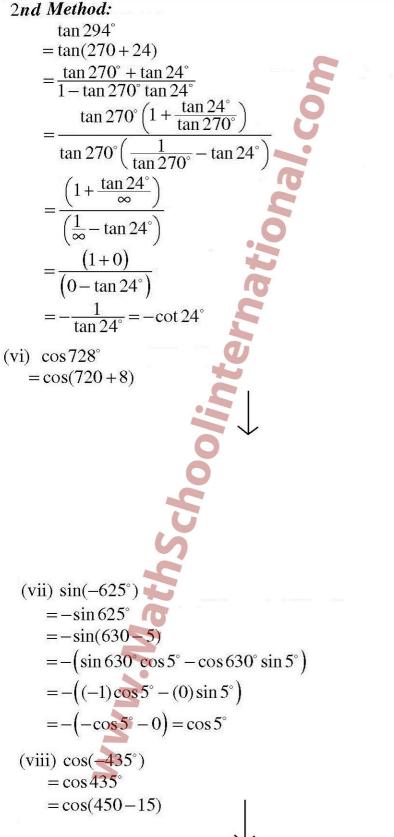
Trigonometric Identities Exercise 10.1 (Solution) for Class XI



Question # 2

Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45°.





Question #3 Prove the following:
(i)
$$\sin(180 + \alpha)\sin(90 - \alpha) = -\sin\alpha\cos\alpha$$
 (iii) $\sin 306^{\circ} + \cos 234^{\circ} + \cos 162^{\circ} + \cos 18^{\circ} = 0$
(ii) $\sin 780^{\circ} \sin 480^{\circ} + \cos 120^{\circ} \sin 30^{\circ} = \frac{1}{2}$ (iv) $\cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} = -1$
Solution
(i) L.I.I.S
 $= \sin(180 + \alpha)\sin(90 - \alpha)$
 $= (\sin 180^{\circ} \cos\alpha + \cos 180^{\circ} \sin\alpha))(\sin 90^{\circ} \cos\alpha - \cos 90^{\circ} \sin\alpha)$
 $= ((0)\cos\alpha + (-1)\sin\alpha))((1)\cos\alpha - (0)\sin\alpha)$
 $= (0 - \sin\alpha))(\cos\alpha - 0)$
 $= -\sin\alpha\cos\alpha = R.H.S$
(ii) L.H.S
 $= \sin 780^{\circ} \sin 480^{\circ} + \cos 120^{\circ} \sin 30^{\circ}$
 $= \sin(2\times 360 + 60)\sin(5\times 90 + 30) + \cos(90 + 30)\sin 30^{\circ}$
 $= \sin 60^{\circ} \cos 30 + (-\sin 30)\sin 30^{\circ}$
 $= (\sqrt{3})(\sqrt{3}) + (-\frac{1}{2})(\frac{1}{2})$
 $= \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = R.H.S$
(iii) L.H.S
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(180 - 18) + \cos 18^{\circ}$
 $= \cos(270 + 36) + \cos(270 - 36) + \cos(270 - 36) + \sin 270\cos 36 + \sin 270\cos 36 + \cos 180^{\circ} \cos 18 + \sin 180^{\circ} \sin 36^{\circ} + \cos 28^{\circ} + \cos 28^{\circ} + \cos 28^{\circ} + \cos 18^{\circ}$
 $= \cos(270 + 36) + \cos(270 - 36) + (-\sin 38) + \cos 18^{\circ}$
 $= \cos(330^{\circ} \sin 56) - \cos(18^{\circ} + \cos 18^{\circ} + \cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 30^{\circ} + \sin 30^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 330^{\circ} \sin 600^{\circ} + \cos 120^{\circ} \sin 150^{\circ} + \cos 330^{\circ} \sin 160^{\circ} + (-\sin 30)^{\circ} (\cos 60^{\circ})$
 $= (\cos 30^{\circ})(-\sin 60) + (-\sin 30)^{\circ} (\cos 60^{\circ})$
 $= (\cos 30^{\circ})(-\sin 60) + (-\sin 30)^{\circ} (\cos 60^{\circ})$
 $= (\cos 30^{\circ})(-\sin 60) + (-\sin 30)^{\circ} (\cos 60^{\circ})$
 $= (\frac{\sqrt{3}}{2}) \left[-\frac{\sqrt{3}}{2}\right] + (\frac{1}{2}) \left(\frac{1}{2}\right)$ ($\frac{\sqrt{3}}{2}$, $\frac{\sqrt{3}}{2}\right] + (\frac{\sqrt{3}}{2}\right] + (\frac{\sqrt{3}}{2}\right]$
 $= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{$

Question # 4 Prove that

$$\frac{\sin^2(\pi + \theta) \tan\left(\frac{3\pi}{2} + \theta\right)}{\sin^2\left(\frac{3\pi}{2} - \theta\right)\cos^2(\pi - \theta)\csc(2\pi - \theta)} = \cos\theta$$
(i) L.H.S

$$=\frac{\sin^2(\pi+\theta) \tan\left(\frac{3\pi}{2}+\theta\right)}{\cot^2\left(\frac{3\pi}{2}-\theta\right)\cos^2(\pi-\theta)\csc(2\pi-\theta)}$$

First we calculate

$$\sin(\pi+\theta) = \sin\pi\cos\theta + \cos\pi\sin\theta$$
$$= (0)\cos\theta + (-1)\sin\theta$$
$$= 0 - \sin\theta = -\sin\theta$$
$$\Rightarrow \sin^{2}(\pi+\theta) = (-\sin\theta)^{2}$$
$$\tan\left(\frac{3\pi}{2}+\theta\right) = \tan\left(3\cdot\frac{\pi}{2}+\theta\right) = \cot\theta \quad \because \quad \frac{3\pi}{2}+\theta \text{ is in the IVth quad}$$
$$\cot\left(\frac{3\pi}{2}-\theta\right) = \cot\left(3\cdot\frac{\pi}{2}-\theta\right) = \tan\theta \quad \because \quad \frac{3\pi}{2}-\theta \text{ is in the IIIrd quad}$$
$$\Rightarrow \cot^{2}\left(\frac{3\pi}{2}-\theta\right) = (\tan\theta)$$
$$\csc(2\pi-\theta) = \csc(-\theta) = -\csc\theta$$
$$\cos(\pi-\theta) = \cos\pi\cos\theta + \sin\pi\sin\theta$$
$$= (-1)\cos\theta + (0)\sin\theta$$
$$= -\cos\theta + 0 = -\cos\theta$$
$$\Rightarrow \cos^{2}(\pi-\theta) = (-\cos\theta)^{2}$$
$$= \frac{(-\sin\theta)^{2}(-\cot\theta)}{(\tan\theta)^{2}(-\cos\theta)^{2}(-\csc\theta)}$$
$$= \frac{\sin^{2}\theta}{(\cos^{2}\theta)}\frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^{2}\theta}{\cos^{2}\theta}\frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta\cos\theta}{\sin\theta}$$
$$= \cos\theta = \text{R.H.S}$$

$$\frac{\text{Question # 4}}{\text{(ii)}} \frac{\text{Prove that}}{\sec(90^\circ + \theta) \sec(-\theta) \tan(180^\circ - \theta)}$$

$$\frac{\cos(90^\circ + \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)}{\sec(360^\circ - \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)} = -1$$

$$\frac{\text{Solution}}{(\text{ii) L.H.S}}$$

$$= \frac{\cos(90^\circ + \theta) \sec(-\theta) \tan(180^\circ - \theta)}{\sec(360^\circ - \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)}$$

First we calculate

 $\cos(90 + \theta) = -\sin\theta$ $\because 90 + \theta \text{ is in the IInd quad.}$ $\sec(-\theta) = \sec\theta$

 $\tan(180 - \theta) = \tan(2(90) - \theta) = -\tan\theta$ $\therefore 180 - \theta$ is in the IInd quad.

 $\sec(360 - \theta) = \sec(-\theta) = \sec\theta$

 $\sin(180 + \theta) = \sin(2(90) + \theta) = -\sin\theta$ $\therefore 180 + \theta$ is in the IIIrd quad.

 $\cot(90 - \theta) = \tan \theta$:: $90 - \theta$ is in the 1st quad.

 $=\frac{(-\sin\theta)\sec\theta(-\tan\theta)}{\sec\theta(-\sin\theta)(-\tan\theta)}$

=1 = R.H.S

= 0 = R.H.S

Question #5 If α, β, γ are the angles of a triangle ABC, then prove that $\cos\left(\frac{\alpha+\beta}{2}\right) = \sin\frac{\gamma}{2}$ $\sin(\alpha + \beta) = \sin \gamma$ (ii) (i) $\tan(\alpha + \beta) + \tan \gamma = 0$ (iii) $\cos(\alpha + \beta) = \cos \gamma$ (iv) Since α , β and γ are angels of triangle therefore Solution $\alpha + \beta + \gamma = 180$ (i)L.H.S = $sin(\alpha + \beta)$ $\alpha + \beta = 180 - \gamma$ $=\sin(180-\gamma)$ $= \sin 180 \cos \gamma - \cos 180 \sin \gamma$ $=(0)\cos\gamma-(-1)\sin\gamma$ $=0+\sin\gamma$ $=\sin \gamma = R.H.S$ L.H.S = $\cos\left(\frac{\alpha+\beta}{2}\right)$ (ii)Since α , β and γ are angels of triangle therefore $\Rightarrow \frac{\alpha + \beta + \gamma = 180}{\alpha + \beta = 180 - \gamma} \Rightarrow \frac{\alpha + \beta}{2} = \frac{180 - \gamma}{2}$ $=\cos\left(\frac{180-\gamma}{2}\right)$ $=\cos\left(\frac{180}{2}-\frac{\gamma}{2}\right)$ $=\cos\left(90-\frac{\gamma}{2}\right)$ $=\cos 90\cos \frac{\gamma}{2} + \sin 90\sin \frac{\gamma}{2}$ $= (0)\cos\frac{\gamma}{2} + (1)\sin\frac{\gamma}{2}$ $=0+\sin\frac{\gamma}{2}$ Since α , β and γ are angels of triangle therefore $=\sin\frac{\gamma}{2}$ = R.H.S $\alpha + \beta + \gamma = 180 \implies \alpha + \beta = 180 - \gamma$ L.H.S = $\cos(\alpha + \beta)$ (iii) $=\cos(180-\gamma)$ $=\cos 180\cos \gamma + \sin 180\sin \gamma$ $=(-1)\cos\gamma+(0)\sin\gamma$ $= -\cos \gamma + 0$ $=-\cos\gamma = R.H.S$ L.H.S = $tan(\alpha + \beta) + tan \gamma$ (iv) $= \tan(180 - \gamma) + \tan \gamma$ $=\frac{\tan 180 - \tan \gamma}{1 + \tan 180 \tan \gamma} + \tan \gamma$ $=\frac{(0)-\tan\gamma}{1+(0)\tan\gamma}+\tan\gamma$ Since α , β and γ are angels of triangle therefore $\alpha + \beta + \gamma = 180 \implies \alpha + \beta = 180 - \gamma$ $=\frac{-\tan\gamma}{1+0}+\tan\gamma$ $= -\tan \gamma + \tan \gamma$