Question # 1 Prove that

(i) $\sin(180^\circ+\theta)=-\sin\theta$ (ii) $\cos(180^\circ+\theta)=-\cos\theta$

(iii) $\tan\left(270^\circ-\theta\right)=\cot\theta$ (iv) $\cos(\theta-180^\circ)=-\cos\theta$

(v) $\cos(270^\circ+\theta)=\sin\theta$ (vi) $\sin(\theta+270^\circ)=-\cos\theta$

(vii) $\tan\left(180^\circ+\theta\right)=\tan\theta$ (viii) $\cos(360^\circ-\theta)=\cos\theta$

Question # 2 Find the values of the following:

(i) $\sin 15^\circ$ (ii) $\cos 15^\circ$

(iii) $\tan 15^\circ$

Question # 3 Prove that:

(i) $\sin(45^\circ+\alpha)=\dfrac{1}{\sqrt{2}}(\sin\alpha+\cos\alpha)$

(ii) $\cos(45^\circ+\alpha)=\dfrac{1}{\sqrt{2}}(\cos\alpha-\sin\alpha)$

Question # 4 Prove that:

(i) $\tan(45+A)\tan(45-A)=1$

(ii) $\tan\left(\dfrac{\pi}{4}-\theta\right)+\tan\left(\dfrac{3\pi}{4}+\theta\right)=0$

(iii) $\sin\left(\theta+\dfrac{\pi}{6}\right)+\cos\left(\theta+\dfrac{\pi}{3}\right)=\cos\theta$

(iv) $\dfrac{\sin\theta-\cos\theta\tan\dfrac{\theta}{2}}{\cos\theta+\sin\theta\tan\dfrac{\theta}{2}}=\tan\dfrac{\theta}{2}$

(v) $\dfrac{1-\tan\theta\tan\varphi}{1+\tan\theta\tan\varphi}=\dfrac{\cos(\theta+\varphi)}{\cos(\theta-\varphi)}$

Question # 5 Show that:

$\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\alpha-\sin^2\beta=\cos^2\beta-\sin^2\alpha$

Question # 6 Show that:

$\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)}=\tan\alpha$

Question # 7 Show that

(i) $\cot(\alpha+\beta)=\dfrac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$

(ii) $\cot(\alpha-\beta)=\dfrac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

(iii) $\dfrac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}=\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

Question # 8 If $\sin\alpha = \dfrac{4}{5}$ and $\cos\alpha = \dfrac{40}{41}$ where $0 < \alpha < \dfrac{\pi}{2}$ and $0 < \beta < \dfrac{\pi}{2}$.

Show that $\sin(\alpha - \beta) = \dfrac{133}{205}$

Question # 9 If $\sin\alpha = \dfrac{4}{5}$ and $\sin\beta = \dfrac{12}{13}$ where $\dfrac{\pi}{2} < \alpha < \pi$ and $\dfrac{\pi}{2} < \beta < \pi$. Find

(i) $\sin(\alpha+\beta)$ (ii) $\cos(\alpha+\beta)$

(iii) $\tan(\alpha+\beta)$ (vi) $\tan(\alpha-\beta)$

(iv) $\sin(\alpha-\beta)$ (v) $\cos(\alpha-\beta)$

In which quadrant do the terminal sides of the angles of measures $(\alpha+\beta)$ and $(\alpha-\beta)$ lie

Question # 10 Find $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$, given that

(i) $\tan\alpha = \dfrac{3}{4}$, $\sin\beta = \dfrac{5}{13}$ and neither the terminal side of the angle of measure α nor that of β is in the I quadrant.

(ii) $\tan\alpha = -\dfrac{15}{8}$, $\sin\beta = -\dfrac{7}{25}$ and neither the terminal side of the angle of measure α nor that of β is in the IV quadrant.

Question # 11 Prove that:

$$\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$$

Question # 12 If α, β, γ are the angles of a tringle ABC, show that

$$\cot\frac{\beta}{2} + \cot\frac{\alpha}{2} + \cot\frac{\gamma}{2} = \cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$$

Question # 13 If $\alpha+\beta+\gamma = 180^\circ$, show that

$\cot\alpha\cot\beta + \cot\beta\cot\gamma + \cot\gamma\cot\alpha = 1$

Question # 14 Express the following in the form $r\sin(\theta+\phi)$ or $r\sin(\theta-\phi)$, where terminal sides of the angles of measure θ and ϕ are in the first quadrant:

(i) $12\sin\theta + 5\cos\theta$ (ii) $3\sin\theta - 4\cos\theta$

(iii) $\sin\theta - \cos\theta$ (iv) $5\sin\theta - 4\cos\theta$

(v) $\sin\theta + \cos\theta$